$$n_0 = k - 1 + (1 - \nu) \ln \left[\frac{(\psi - \psi_1)_{r=r_1}}{(\psi - \psi_1)_{r=r_2}} \frac{\varphi_{r=r_2}}{\varphi_{r=r_1}} \right] \Big/ \ln \left(\frac{r_1}{r_2} \right); \qquad (14)$$

$$a_0 = \frac{r_1^{n_0 - l_0 + 2}}{(n_0 - l_0 + 2)^2 d_{r=r_1}} = \frac{r_2^{n_0 - l_0 + 2}}{(n_0 - l_0 + 2)^2 d_{r=r_2}}. \qquad (15)$$

It can be shown that for other relative locations of the measurement points as well, and furthermore, for other laws of variation of λ and c with respect to the spatial variable, we can construct exact explicit functions of various degrees of complexity. The essence of the method of construction of such functions consists in finding in three-dimensional space some Laplace mappings of differential equations in the transform parameter, one of whose particular solutions is the relation of the mappings of the temperatures at some points in terms of a coordinate which in turn is expressed by means of some special functions whose arguments contain the desired coefficients.

NOTATION

τ, $\bar{\tau}$, ϑ, time; r, coordinate; T, temperature; λ, thermal conductivity; c, volumetric heat capacity.

LITERATURE CITED

1. Yu. L. Gur'ev and D. N. Chubarov, "Determination of the thermal conductivities and thermal diffusivities of materials on the basis of measurements of nonstationary temperatures," Inzh.-Fiz. Zh., 35, No. 2, 250-257 (1978).
2. V. I. Zhuk, S. A. Il'in, and D. N. Chubarov, "Identification of the constants of heat and mass transfer from the data of measurements made in nonstationary regimes," Inzh.-Fiz. Zh., 41, No. 2, 225-231 (1981).
3. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).

NONSTATIONARY HEAT TRANSFER BY THE METHOD OF SOLVING THE INVERSE HEAT-CONDUCTION PROBLEM

I. M. Lagun

UDC 536.244

The heat-transfer coefficient between a gas and a solid under nonstationary conditions is investigated and computational dependences are obtained.

In multimode, pulse power plants of short operating time an important role is played by transients that are characterized by a gasdynamic and thermal nonstationarity. As investigations showed, the thermogasdynamic nonstationarity can be manifest during a large or even the whole operating time.

The thermal state of a construction using transition regimes has been studied inadequately. The mean temperatures, as well as the temperature fields determined by means of the averaged parameters, do not reflect the features of the transients and do not satisfy the requirements of practice [1].

In the general engineering [2, 3] and specialized [4, 5] purpose papers, the heat-transfer coefficient is averaged, as a rule. Insertion of the quasistationary heat-transfer coefficients does not reduce the problem of nonstationary heat transfer. Unfortunately, there are no relationships on the change in the heat-transfer coefficient in time under transient heat-transfer conditions.

Tula Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 5, pp. 797-802, November, 1983. Original article submitted February 2, 1983.

0022-0841/83/4505-1289$07.50

The purpose of this paper is the experimental-theoretical investigation of the heat-transfer coefficient under nonstationary conditions that are characteristic for transient regimes by the method of solving the inverse heat-conduction problem (IHCP). Underlying the paper is an experimental study of the heat transfer in the start-up period of heat-exchanger operation.

The experimental investigations were performed on a thermal power plant with a broad range of flux velocity (M = 0-0.3) and gas pressure (p = 5-25 MPa) changes in the heat exchanger. The nonstationary process of heat transmission through the wall under nonsymmetric conditions of heat transfer to a "hot" (T_h, α_h) and a "cold" (T_c, α_c) gas was studied here. The heat-transmission process was almost one-dimensional.

As a result of the experimental investigations, temperature curves $T = f(\tau)$ were obtained for fixed points of the construction which were removed by a quantity $x = x_1$ from the heated surface while in the zone of reliable thermometry. The size of the zone mentioned was set up for the nonstationary case.

The solution of the IHCP by determining the nonstationary heat-transfer coefficient was executed on a specialized electrical modeling apparatus [6]. The nonlinear heat-conduction equation with variable boundary conditions of the third kind

$$c\rho \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right), \quad 0 < x < \delta, \ \tau > 0;$$

$$T = T_i, \quad 0 \leqslant x \leqslant \delta, \ \tau = 0;$$

$$T = f(\tau), \quad x = x_1, \ \tau > 0;$$

$$\lambda \frac{\partial T}{\partial x} + \alpha_c (T_c - T) = 0, \quad x = \delta, \ \tau > 0;$$

$$T = T_h, \quad x < 0, \ \tau \geqslant 0$$

was solved here on an electrical model of resistors and capacitors (RC-model).

The solution of the inverse problem on the "hot" boundary was by the method in [7]. The criterion for the estimate was based on agreement between the experimental $T = f(\tau)$ and the temperature curve obtained on the electrical model in time at the fixed point $x = x_1$ of the heat-exchanger wall. The magnitude of the residual caused by both the errors in the calculations $\alpha_h = f(\tau)$ and the nonlinearity was found for a number of characteristic times, and determined the accuracy of the solution.

The desired time dependence of the heat-transfer coefficient was also determined by performing a thermal experiment with a measurement of the thermal flux density, the gas temperature, and the wall surface.

The discrepancy in determining α_h by the method of solving the IHCP and by the thermal experiment did not exceed (9-10)%, where the greatest error was just in the initial section of the research while the error was considerably less during the main, greater time of the transient regime.

Results of solving the IHCP and the thermal experimental investigation of the heat-transfer coefficient in transient operating regimes are represented in Fig. 1 for different pressures and in different heat-exchanger sections. The pressure in the heat exchanger was here 7, 14, 23 MPa and the sections were selected with weakly (a) and strongly (b) developed turbulence. Points on this same figure denote the thermal experiment data.

The investigations represented show that the heat-transfer coefficient varies 2-8 times in different sections and for different gas pressures in the heat exchanger. The investigations we performed on the RC-models showed that not taking account of the above-mentioned changes results in deviations in the determination of the structure temperature of up to (30-50)%, especially in the initial stage of operation.

Taking into account that the heat-transfer coefficient in the heat exchanger diminishes from the maximal to the quasi- or stationary value, we assume that the relative heat-transfer coefficient $\bar{\alpha}_h$ can be approximated by the dependence $\alpha_h = k_1 t^m + k_2$, where $\bar{\alpha}_h = (\alpha_h - \alpha_0)/(\alpha_m - \alpha_0)$; $t = \tau/\tau_t$.

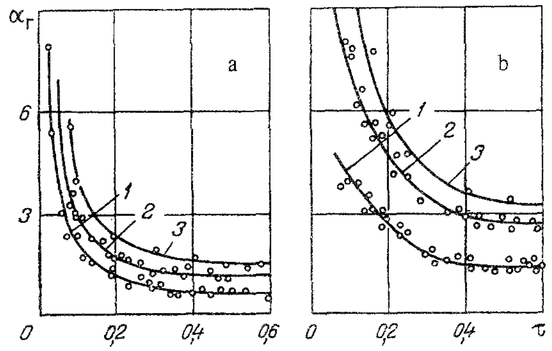

Fig. 1. A change in the heat-transfer coefficient in the heat exchanger with a weakly (a) and strongly (b) developed turbulence for different pressures: 1) p = 7 MPa; 2) 14; 3) 23; α_h, kW/m^2·K; τ, sec.

TABLE 1. Relative Heat-Transfer Coefficient

m	*t*: 0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
0	—	0	0	0	0	0	0	0	0	0	0
0,05	1,0	0,1087	0,0773	0,0584	0,0448	0,0341	0,0252	0,0177	0,0111	0,0053	0
0,10	1,0	0,2057	0,1487	0,1134	0,0876	0,0670	0,0498	0,0350	0,0221	0,0105	0
0,15	1,0	0,2921	0,2145	0,1652	0,1284	0,0987	0,0738	0,0521	0,0329	0,0157	0
0,20	1,0	0,3690	0,2752	0,2140	0,1674	0,1294	0,0971	0,0688	0,0436	0,0209	0
0,25	1,0	0,4377	0,3313	0,2599	0,2047	0,1591	0,1199	0,0853	0,0543	0,0260	0
0,30	1,0	0,4988	0,3830	0,3032	0,2403	0,1877	0,1421	0,1015	0,0648	0,0311	0
0,35	1,0	0,5533	0,4307	0,3439	0,2744	0,2154	0,1637	0,1174	0,0751	0,0362	0
0,40	1,0	0,6019	0,4747	0,3822	0,3069	0,2421	0,1848	0,1330	0,0854	0,0413	0
0,50	1,0	0,6838	0,5528	0,4523	0,3675	0,2929	0,2254	0,1633	0,1056	0,0513	0
0,60	1,0	0,7488	0,6193	0,5144	0,4229	0,3402	0,2640	0,1927	0,1253	0,0613	0
0,70	1,0	0,8005	0,6759	0,5695	0,4734	0,3844	0,3006	0,2209	0,1445	0,0711	0
0,80	1,0	0,8415	0,7241	0,6183	0,5194	0,4257	0,3355	0,2492	0,1635	0,0808	0
0,90	1,0	0,8741	0,7651	0,6616	0,5616	0,4641	0,3886	0,2766	0,1819	0,0905	0
1,00	1,0	0,9000	0,8000	0,7000	0,6000	0,5000	0,4000	0,3000	0,2000	0,1000	0

The constant coefficients k_1 and k_2 are determined from the boundary conditions: for t = 0 we have $\alpha_h = \alpha_m$, and $\alpha_h = \alpha_0$ for t = 1. Therefore, we finally obtain

$$\bar{\alpha}_h = 1 - t^m. \tag{1}$$

The relative heat-transfer coefficient can be represented in the criterial form

$$\bar{\alpha}_h = \frac{\mathrm{Nu} - \mathrm{Nu}_0}{\mathrm{Nu}_m - \mathrm{Nu}_0}. \tag{2}$$

Since the relative time t and the order of the parabola vary between zero and one, then the relative heat-transfer coefficient is easily tabulated. Such tables, facilitating the computation of the heat-transfer coefficient, have been compiled, and we present one of the modifications (see Table 1).

To determine the true value of the heat-transfer coefficient we obtain the dependence

$$\alpha_h = \alpha_0 + (\alpha_m - \alpha_0)(1 - t^m). \tag{3}$$

It follows from (1) and (3) that the nature of the change in the heat-transfer coefficient depends to a substantial extent on the order of the parabola, which varies between zero and one.

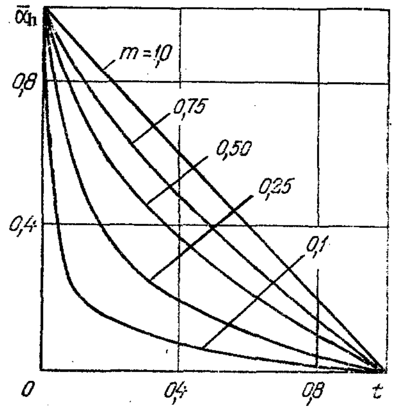

Fig. 2

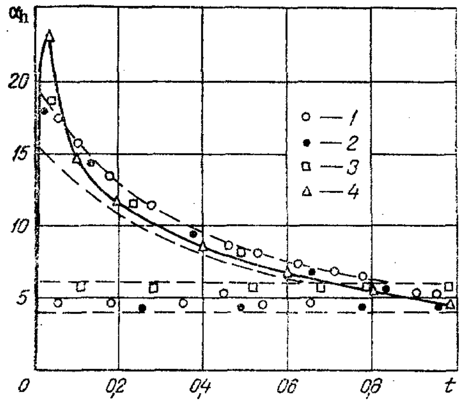

Fig. 3

Fig. 2. Approximation of the heat-transfer coefficient; $\bar{\alpha}_h$, t are relative (dimensionless) quantities.

Fig. 3. Values of the heat-transfer coefficient: solid line from (1); 1) from formula (4); 2) from (5); 3) from (6); 4) experimental data; dashed lines depict the zone of data spread; t is the relative time.

Analysis of the data obtained permits the assertion that the results of experiments can always be approximated by a parabola of m order. The order of the parabola will here differ for different pressures in the chamber. The nature of the change in the heat-transfer coefficient is shown in Fig. 2 as a function of the order of the parabola, from which it follows that as m approaches zero or one the parabola degenerates into straight lines.

It is experimentally established that as the pressure rises the exponent m increases. This is explained by the fact that as the pressure rises the heat-exchange regime is stabilized to a great degree.

Recommendations on the selection of the exponent m for the pressures 5-25 MPa are given on the basis of experimental investigations as m = 0.08-0.25. Smaller values of m correspond to lower pressures.

It is established that the heat-transfer coefficient in the transient heat-exchange regimes can differ 2-8 times from the stationary value. These changes are explained by deviations of the parameters of the nonstationary heat- and mass-transfer process from the steady-state values in the boundary layer. The deviations of the heat-transfer coefficient from the stationary values diminish with boundary-layer stabilization and take on stationary values in the limit. The time of stabilization of the thermal-gasdynamic circumstances in the boundary layer can be determined in a first approximation by the rate of progress of the transients.

The proposed dependence (1) is compared with the known criterial relationships for forced heat exchange [2, 3, 5, 8] to estimate the confidence of the results obtained:

$$\mathrm{Nu} = 0.023\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}\,\beta, \tag{4}$$

$$\mathrm{Nu} = 0.0225\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}, \tag{5}$$

$$\mathrm{Nu} = 0.029\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}\,(T_h/T)^{0.39}. \tag{6}$$

Results of the comparison are shown in Fig. 3. Given on this same figure are the stationary and quasistationary values, and the experimental values of the heat-transfer coefficient are also presented.

It follows from the results obtained that the dependence developed contradicts that known in the domain of stationary heat exchange, and extends the boundary of the heat-transfer coefficient computation to the case of nonstationary heat exchange.

Let us consider the method of computing the nonstationary heat-transfer coefficient in an example of the transient operating regime of a thermal motor. We take the period of motor start-up as the transient regime.

The whole time of motor operation is divided into three periods. During the first period of the emergence into the regime we consider the heat-transfer coefficient to vary between 0 and the maximal value α_m and its growth is proportional to the growth of time:

$$\alpha_h = \alpha_m t \quad \text{for } 0 \leqslant t \leqslant \tau/\tau_1. \tag{7}$$

The maximal value of the heat-transfer coefficient α_m is found from the known stationary value:

$$\alpha_m = k_\alpha \alpha_0. \tag{8}$$

It is established that under nonstationary conditions the coefficient of nonstationary heat exchange varies between the limits k_α = 2-8. For approximate computations it is possible to take k_α = 5.0.

In the second nonstationary heat-exchange period, the heat-transfer coefficient drops from the maximal value at the beginning to the stationary value at the end of the period. To a sufficient degree of accuracy this change can be approximated by the dependences (1) or (3).

And lastly is the period of steady-state heat exchange in which the change in the heat-transfer coefficient can be neglected while the coefficient itself $\alpha_h = \alpha_0$ is computed from the known criterial dependences (4), (5), (6).

The investigations performed and the results obtained show that the heat-transfer coefficient undergoes substantial two- to eightfold changes in the transient regimes, which should be taken into account in performing the thermal computations; on the basis of experimental data, the proposed method of computing the heat-transfer coefficient extends the possibility of computing the nonstationary heat transfer, is simple and convenient for engineering utilization.

NOTATION

T, temperature; T_h and T_c, temperature of the "hot" and "cold" gas; α_h and α_c, heat-transfer coefficients between the heat-exchanger and the "hot" and "cold" gas; α_m and α_0, maximal and stationary heat-transfer coefficient; p, pressure; M, Mach number; τ, time; τ_t, total operating time; τ_1, time of emergence into the regime; k_α, coefficient of nonstationarity; δ, wall thickness; λ, heat-conduction coefficient; c, specific heat; Nu_0 and Nu_m, stationary and maximal Nusselt criteria, respectively; and Re and Pr, Reynolds and Prandtl numbers.

LITERATURE CITED

1. I. M. Lagun, "Investigation of the thermal regime of a power plant structure under variable heat exchange parameters," Heat Exchange and Modeling in Power Plants, Abstracts of Reports of an All-Union Conference [in Russian], Tula (1979), pp. 77-78.
2. S. S. Kutateladze, Principles of the Theory of Heat Transfer [in Russian], Mashgiz, Moscow (1962).
3. M. A. Mikheev, Principles of Heat Transfer [in Russian], Gosénergoizdat (1956).
4. E. V. Kudryavtsev, K. I. Chakalev, and N. V. Shumakov, Nonstationary Heat Transfer [in Russian], Izd. Akad. Nauk SSSR, Moscow (1961).
5. V. K. Koshkin (ed.), Principles of Heat Transfer in Aviation and Space-Rocket Engineering [in Russian], Mashgiz, Moscow (1975).
6. M. P. Kuz'min, Electrical Modeling of Nonstationary Heat Transfer Processes [in Russian], Énergiya, Moscow (1974).
7. M. P. Kuz'min, I. M. Lagun, and S. V. Lagun, "Solution of inverse heat-conduction problems on specialized analog computers," Inzh. Fiz. Zh., 33, No. 6, 1125-1130 (1977).
8. V. K. Koshkin et al., Nonstationary Heat Transfer [in Russian], Mashinostroenie, Moscow (1973).